**Set Theory**

**A set is a well-defined collection of objects or elements**.

Each element in a set is unique. Usually, but not necessarily, a set is denoted by a capital letter e.g., A, B, .. etc. and the elements are enclosed between brackets { }.

***Examples:***  
   
A = Set of all small English alphabets

   = {a, b, c, ....., x, y, z}

B = Set of all positive integers less than or equal to 10

   = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

Properties of the Elements:

* Elements are well defined
* Elements are unique eg {1,2,3} = {1,1,3,2} = {2,1,3,3}

For the representation of a set the following three methods are commonly used:

(i) Statement form method

(ii) Roster or tabular form method

(iii) Rule or set builder form method

1. Statement form:

In this, well-defined description of the elements of the set is given and the same are enclosed in curly brackets.

**For example:**

(i) The set of odd numbers less than 7 is written as: **{odd numbers less than 7}.**  
  
(ii) A set of football players with ages between 22 years to 30 years.

(iii) A set of numbers greater than 30 and smaller than 55.

2. Roster form or tabular form:

In this, elements of the set are listed within the pair of brackets { } and are separated by commas.

**For example:**

(i) Let N denote the set of first five natural numbers.

Therefore, N = {1, 2, 3, 4, 5}          
  
(ii) The set of all vowels of the English alphabet.

Therefore, V = {a, e, i, o, u }  
  
 (iii) The set of all letters in the word MATHEMATICS.

Therefore, Z = {M, A, T, H, E, I, C, S}

3. Set builder form:

In this, a rule, or the formula or the statement is written within the pair of brackets so that the set is well defined. In the set builder form, all the elements of the set, must possess a single property to become the member of that set.   
  
In this form of representation of a set, the element of the set is described by using a symbol ‘x’ or any other variable followed by a colon The symbol ‘:‘ or ‘|‘ is used to denote such that and then we write the property possessed by the elements of the set and enclose the whole description in braces. In this, the colon stands for ‘such that’ and braces stand for ‘set of all’.   
  
**For example:**

(i) Let P is a set of counting numbers greater than 12;  
the set P in set-builder form is written as :

                P = {x : x is a counting number and greater than 12}  
                                                 **or**  
                P = {x | x is a counting number and greater than 12}

This will be read as, 'P is the set of elements x such that x is a counting number and is greater than 12'.

**Example**

The set of integers lying between -2 and 3.   
  
**Statement form:** {I is a set of integers lying between -2 and 3}   
  
**Roster form:** I = {-1, 0, 1, 2}   
  
**Set builder form:** I = {x : x ∈ I, -2 < x < 3}

**Definitions**

The empty set is a set containing no objects. It is written as a pair of curly braces with nothing inside {} or by using the symbol ∅.

**The universal set**, at least for a given collection of set theoretic computations, is the set of all possible objects.

The set membership symbol ∈ is used to say that an object is a member of a set. It has a partner symbol ∈/ which is used to say an object is not in a set.

We say two sets are equal if they have exactly the same members.

**The cardinality of a set** is its size. For a finite set, the cardinality of a set is the number of members it contains. In symbolic notation the size of a set S is written |S|. For the set S = {1, 2, 3, 2} we show cardinality by writing |S| = 3.

**The compliment of a set S** is the collection of objects in the universal set that are not in S. The compliment is written S c . In curly brace notation S c = {x : (x ∈ U) ∧ (x /∈ S)} or more compactly as S c = {x : x /∈ S}

**What are the four basic operations on sets?**

**Solution:**

The four basic operations are:

**1.** Union of Sets

**2.** Intersection of sets

**3.** Complement of the Set

**4.** Cartesian Product of sets

**The union of two sets** S and T is the collection of all objects that are in either set. It is written S ∪ T . Using curly brace notion S ∪ T = {x : (x ∈ S) or (x ∈ T )}

Suppose S = {1, 2, 3}, T = {1, 3, 5}, and U = {2, 3, 4, 5}. Then:

S ∪ T = {1, 2, 3, 5}, S ∪ U = {1, 2, 3, 4, 5}, and T ∪ U = {1, 2, 3, 4, 5}

**The intersection of two sets** S and T is the collection of all objects that are in both sets. It is written S ∩ T . Using curly brace notation S ∩ T = {x : (x ∈ S) and (x ∈ T )} The symbol and in the above definition is an example of a Boolean or logical operation.

Suppose S = {1, 2, 3, 5}, T = {1, 3, 4, 5}, and U = {2, 3, 4, 5}.

Then: S ∩ T = {1, 3, 5}, S ∩ U = {2, 3, 5}, and T ∩ U = {3, 4, 5}

**The difference of two sets** S and T is the collection of objects in S that are not in T . The difference is written S − T .

In curly brace notation S − T = {x : x ∈ (S ∩ (T c ))}, or alternately S − T = {x : (x ∈ S) ∧ (x /∈ T )}

**Venn Diagrams**

A Venn diagram is a way of depicting the relationship between sets. Each set is shown as a circle and circles overlap if the sets intersect.

The following are Venn diagrams for the intersection and union of two sets. The shaded parts of the diagrams are the intersections and unions respectively.

 

A Complement



**The difference of two sets S and T** is the collection of objects in S that are not in T . The difference is written S − T . In curly brace notation S − T = {x : x ∈ (S ∩ (T c ))}, or alternately S − T = {x : (x ∈ S) ∧ (x /∈ T )}

**Subsets**

For two sets S and T we say that S is a subset of T if each element of S is also an element of T. In formal notation S ⊆ T if for all x ∈ S we have x ∈ T.

If S ⊆ T then we also say T contains S which can be written T ⊇ S. If S ⊆ T and S 6= T then we write S ⊂ T and we say S is a proper subset of T.

**Example** : If A = {a, b, c} then A has eight different subsets: ∅ {a} {b} {c} {a, b} {a, c} {b, c} {a, b, c} Notice that A ⊆ A and in fact each set is a subset of itself. The empty set ∅ is a subset of every set.

**De Morgan’s Laws**: Suppose that S and T are sets. DeMorgan’s Laws state that

1. (S ∪ T ) c = S c ∩ T c , and
2. (S ∩ T ) c = S c ∪ T c .

**Problems**

1. Which of the following are sets? Assume that a proper universal set has been chosen and answer by listing the names of the collections of objects that are sets.

* B = {A, E, I, O, U}
* C = { √ x : x < 0}
* D = {1, 2, A, 5, B, Q, 1, V }
* F is a list of the weight, to the nearest kilogram, of all people that were in Mumbai at any time in 2017

1. Suppose that we have the set U = {n : 0 ≤ n < 100} of whole numbers as our universal set. Let P be the prime numbers in U, let E be the even numbers in U, and let F = {1, 2, 3, 5, 8, 13, 21, 34, 55, 89}. Describe the following sets either by listing them or with a careful English sentence.

(i) Ec , (ii) P ∩ F, (iii) P ∩ E, (iv) F ∩ E ∪ F ∩ Ec , and (v) F ∪ F c .

3. Suppose that we take the universal set U to be the integers. Let S be the even integers, let T be the integers that can be obtained by tripling any one integer and adding one to it, and let V be the set of numbers that are whole multiples of both two and three.

(i) Write S, T , and V using symbolic notation.

(ii) Compute S ∩ T , S ∩ V and T ∩ V and give symbolic representations that do not use the symbols S, T , or V on the right hand side of the equals sign.

1. Compute the cardinality of the following sets. You may use other texts or the internet.
2. Two digit positive odd integers.
3. Elements present in a sucrose molecule.
4. Isotopes of hydrogen that are not radioactive.
5. Planets orbiting the same star as the planet you are standing on that have moons. Assume that Pluto is a minor planet.
6. Elements with seven electrons in their valence shell. Remember that Ununoctium was discovered in 2002 so be sure to use a relatively recent reference.
7. Subsets of S = {a, b, c, d} with cardinality 2.
8. Prime numbers whose base-ten digits sum to ten. Be careful, some have three digits.
9. Find an example of an infinite set that has a finite complement, be sure to state the universal set.
10. Find an example of an infinite set that has an infinite complement, be sure to state the universal set.
11. Give the Venn diagrams for the following sets.

(i) A – B (ii) B – A (iii) Ac ∩ B (iv) A ⊕B (v) (A ⊕B) c (vi) Ac ∪ Bc

8. Suppose we take the universal set to be the set of non-negative integers. Let E be the set of even numbers, O be the set of odd numbers and F = {0, 1, 2, 3, 5, 8, 13, 21, 34, 89, 144, ...} be the set of Fibonacci numbers. The Fibonacci sequence is 0, 1, 1, 2, 3, 5, 8, . . . in which the next term is obtained by adding the previous two.

(i) Prove that the intersection of F with E and O are both infinite.

(ii) Make a Venn diagram for the sets E, F, and O, and explain why this is a Mickey-Mouse problem.

1. A binary operation ⊙ is commutative if x ⊙ y = y ⊙ x. An example of a commutative operation is multiplication. Subtraction is noncommutative. Determine, with proof, if union, intersection, set difference, and symmetric difference are commutative.
2. A) Prove that A ∪ (B ∪ C) = (A ∪ B) ∪ C

B) Prove that A ∩ (B ∩ C) = (A ∩ B) ∩ C

c) Prove that A ⊕ (B ⊕ C) = (A ⊕B) ⊕C

D) Disprove that A ⊕ (B ∪ C) = (A ⊕ B) ∪ C

10. Consider the set S = {1, 2, 3, 4}. For each k = 0, 1, . . . , 4 how many k element subsets does S have?

11. Suppose we have a set S with n ≥ 0 elements. Find a formula for the number of different subsets of S that have k elements.

12. For finite sets S and T , prove |S ∪ T | = |S| + |T | − |S ∩ T |